MERENJE FREKVENCIJE 633 nm JODNO STABILISANOG He-Ne LASERA POMOĆU OPTIČKOG FEMTOSEKUNDNOG KOMB GENERATORA

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Кључне речи: komb generator, etalon dužine, jodno stabilisani laser, metrologija optičkih frekvencija

КРАТАК САДРЖАЈ

Pojavom komb generatora napravljen je ogroman napredak u metrologiji optičkih frekvencija eliminišući potrebu za komplikovanim frekvencijskim lancima. Nabavkom komb generatora DMDM je ostvario novu praktičnu realizaciju nacionalnog etalona jedinice dužine, metra. U radu su prikazani rezultati merenja frekvencije stare realizacije nacionalnog etalona dužine, jodno stabilisanog He-Ne lasera,pomoću novog etalona SI metra, komb generatora. Merenja su izvršena pomoću metode izbijanja optičkih frekvencija između komb generatora i jodno stabilisanog He-Ne lasera. Rezultati su pokazali dobro slaganje sa rezultatima dobijenim u ključnom poređenju CCL-K11 i CCL preporučenom vrednošću unutar preporučene merne nesigurnosti od 20 kHz.

ABSOLUTE FREQUENCY MEASUREMENT OF THE 633 nm IODINE STABILIZED He-Ne LASER BY OPTICAL FEMTOSECOND COMB GENERATOR

Keywords: frequency comb generator, length standard, iodine stabilized laser, optical frequency metrology

ABSTRACT

The appearance of the frequency comb generators has enormously revolutionized the metrology of optical frequencies, eliminating the need for complicated frequency chains. By the new frequency comb generator, DMDM established a new way of practical realization of the National standard of length, the metre. In this paper we present the results of absolute frequency measurement of the old realization of the national standard of length, iodine stabilized He-Ne laser, by using new realization, frequency comb generator. The measurements are made by beat frequency method between frequency comb generator and iodine stabilized He-Ne laser. The results are shown good agreement with results obtained in key comparison CCL-K11 and with CCL recommended value within recommended uncertainty of 20 kHz.

INTRODUCTION

The International System of Units (SI) has been updated and refined over many decades since its foundations were laid at the end of the 18th and beginning of the 19th centuries, culminating in the first formalisation of the SI at the 11th General Conference on Weights and Measures (CGPM) in 1960. Following recent advances in measurement of fundamental constants, the SI was revised in May 2019, leading to the current SI that is based on a set of seven defining constants, drawn from the fundamental constants of physics and other constants of nature, which is described in the 9th edition of the SI brochure [1].Accordingly, the supporting documents for the practical realization of the SI units, *mises en pratique* (MeP) [2], were adapted to reflect the revision of the SI and were also published on May 20, 2019.

Metre definition

The definition of the metre, SI base unit of length, is as follows [1]:
The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum *c* to be 299 792 458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.
This definition ensures continuity of the SI base unit of length and implies that ‘the metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second’, as stated in the previous definition of the metre from 1983.
The fundamental equation underlying the above definition of the metre is a direct relationship between a length, a time interval and the speed of light:

$$l = c \cdot \Delta t \qquad (1)$$

in which c is the fixed value for the speed of light in vacuum, c = 299 792 458 m s^{-1}, and Δt is the travelling time of the light along a geometrical path, of length l. Realization of the length unit, at a primary level, is thus linked to measurement of light travelling time.

PRACTICAL REALIZATION

List of standard frequencies

Various sources of radiation have been recommended as standards of wavelength and have been updated by the CIPM over time. Someone may use one of the values from the list of ‘recommended values of standard frequencies for applications including the practical realization of the metre and secondary representations of the second’. This list, now known as the CIPM (International Committee for Weights and Measures) *List of recommended frequency standard values* [3] is updated periodically by recommendation of new candidate standard frequencies by the Consultative Committee for Length (CCL) and/or Consultative Committee for Time and Frequency (CCTF). Candidate frequencies are examined according to a published set of guidelines and procedures [4] and only those that pass the necessary checks, are recommended to the CIPM for entry into the list.
The list contains specifications relating to each frequency standard which are displayed after selecting a particular standard on the BIPM web page. For the full list of specifications, reference should be made to the original *CIPM Recommendation* (cited in the online list) and to the various updates that have since been approved by the CIPM.
Laboratories which use a light source which is part of the *CIPM List of recommended frequency standard values* for their realisation of the metre are required to take part in the international key comparison CCL-K11 [5] at least every 10 years (unless they are node laboratories in this comparison). The comparison

tests the laboratory's ability to realise the relevant optical frequency standard within their stated uncertainties.
Among the listed optical frequency standards recommended by CIPM, molecular iodine (I_2) holds a unique position in the list and offers a number of reference lines that have been most widely used for metrological calibration [3]. The system at 633 nm has the advantage of being one of the better quality standards based on I2.

DMDM primary laser

One of the most important recommended radiations in the field of length metrology and worldwide precision measurements is that at 474 THz (633 nm) from a He–Ne laser, stabilized on a saturated absorption hyperfine component in iodine molecule 127I2. DMDM laser can be stabilized on seven iodine components in hyperfine spectrum (*d, e, f, g, h, i* and *j*) which are shown on Fig. 1. This laser is used in many laboratories and national metrology institutes (NMI) around the world as an optical frequency standard for practical realization of the SI metre and is commonly used for calibrating the frequency of lasers employed in length measurement (laser interferometers for length measurements).
Although the new types of lasers have appeared in the field of the frequency/wavelength metrology, the HeNe lasers are still common standards of optical frequency. They are relatively simple and robust devices, and their performances are still acceptable for most applications.

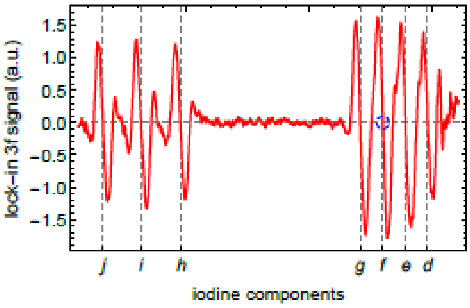

Fig. 1: In the Figure 1 we see third derivative of the seven components of the I_2 spectrum

Block diagram of the primary length standard of the Republic of Serbia [13] is shown in Fig 2, HeNe tube and I_2 cell have windows at Brewster angle. The control unit, based on the two Arduino boards, is standalone device integrated into laser head. The control unit drives both cavity mirrors. Stabilization of the laser's wavelength is based on the signal detected by photodiode beyond high reflectivity mirror. Temperature of the iodine cell is maintained using Peltier element.
Connection to the computer is established by USB. Windows laptop computer is used as graphical user interface (GUI) only: it transmit user commands, displays and records data, but it doesn't perform any direct control on the standard itself. GUI software is written in Python programming language.

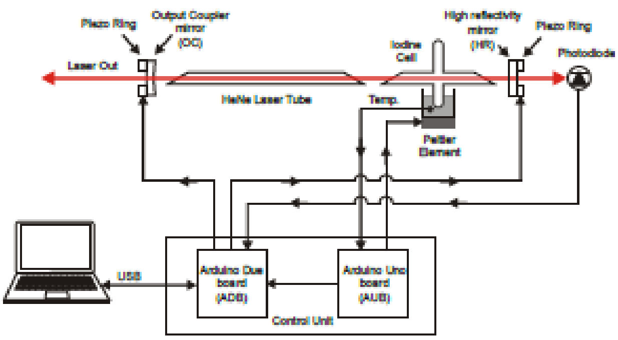

Fig 2. Block diagram of the DMDM primary laser

DMDM comb generator

Length measurements are linked to the SI-second through the definition of the Metre, which fixes the speed of light. Thus, the frequency of a monochromatic light source used in interferometric length measurements must be traceable to the primary frequency standard, the Cs-atomic clock. Until recently, this was achieved using the recommended radiations listed in the Practical Realization of the Definition of the Metre, MeP, which are known at stated uncertainty. The optical frequency metrology was, however,

revolutionized by the invention of the optical frequency comb generators that enables direct absolute measurement of any frequency within the comb range.
So, tracing dimensional metrology measurements back to the SI definition of the metre is nowadays done with the help of optical femtosecond frequency comb generators. These instruments are used to calibrate the frequency of stabilized laser sources. This calibration constitutes the first step in the traceability chain for interferometric measurements and it is offered as a service to the public by many NMIs
The main application of the frequency comb generator at the DMDM is in calibration and performance verification of iodine-stabilized lasers that serve as frequency standards for length metrology. The frequency comb generator provides a realization of the definition of the Metre with very low uncertainty at the optical frequency.
With this system (frequency comb), DMDM is establishing a new practical realization of the metre with improved accuracy of two orders of magnitude (i.e. $2x10^{-13}$ instead of $2,1x10^{-11}$) with respect to the current system based on iodine stabilized lasers.

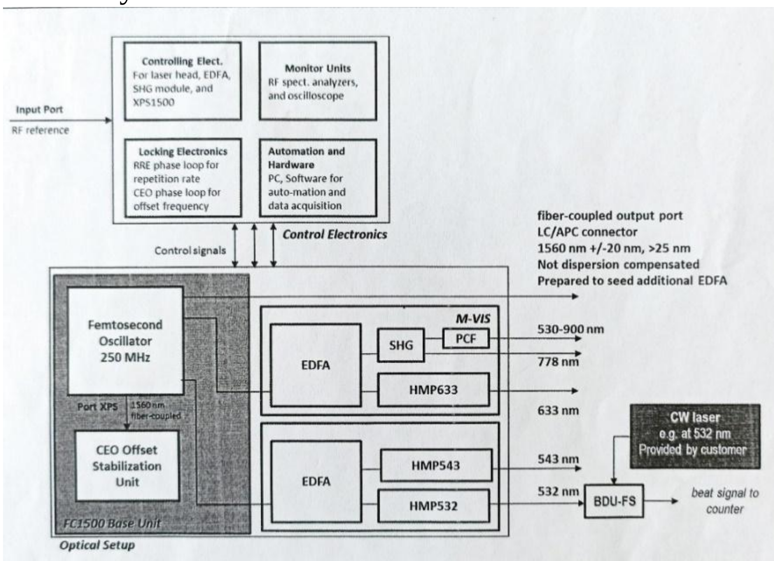

Fig 3. DMDM comb overview (schematic)

DMDM has the optical frequency comb generator manufactured by Menlo Systems, model FC1500 (Fig. 3). This comb is based on erbium-doped, polarization mode locked, femtosecond fibre-ring laser. The comb offers reference frequencies with comb modes of 250 MHz spacing. The repetition rate frequency (250 MHz), f_r, and the carrier-envelope offset frequency (20 MHz), f_o, were referenced to the DMDM primary time and frequency standard, a Cs atomic clock.
The DMDM comb is equipped with several output ports which emit laser radiation at different wavelengths (in nm): 532, 543, 633, 778 and supercontinuum in range (530-900).
A laser radiation emitted by comb is fully determined by the laser repetition rate, f_r, which represents the spacing of the comb lines, and the carrier envelope-offset, f_o, which defines the combs offset from zero. As both f_r and f_o are in radio frequency regime, they may be detected and counted by using standard electronics. Therefore, the comb modes are expressed as

$$f_n = nf_r + f_o \tag{2}$$

with a large ($\approx 10^6$) integer n. This equation maps two radiofrequencies f_r and f_o onto the optical frequencies f_n.
To achieve a stabilized frequency comb two free parameters, i.e. the comb spacing and the comb offset have to be stabilized. It is done by referenced them to (and synchronized with) the DMDM primary time and frequency standard, a Cs atomic clock.
The fibre oscillator consists of the laser head with internal doped fibre amplifier EDFA centered at 1560 nm with power up to 2 mW. The femtosecond laser output power is split and fed to the monitor port and the external parts of the system. One branch is amplified in an external EDFA and spectrally broadened in a highly nonlinear fibre (HNLF) to cover a spectrum of one octave in frequency space. f_0 then is detected by the self-referencing technique with an $f - 2f$ interferometer setup, by taking the difference of the frequencies of a comb mode at f_n and the second harmonic of the mode at f_{2n} (Fig. 4). The other branches are used for generating high power IR light at 1560 nm for subsequent frequency doubling at 780 nm

radiation, for a several output ports which emit laser radiation at different wavelengths (in nm): 532, 543, 633, 778 and for a photonic crystal fibre setup for subsequent broadening of the second harmonic generation (SHG) output to supercontinuum in range (530-900) nm.

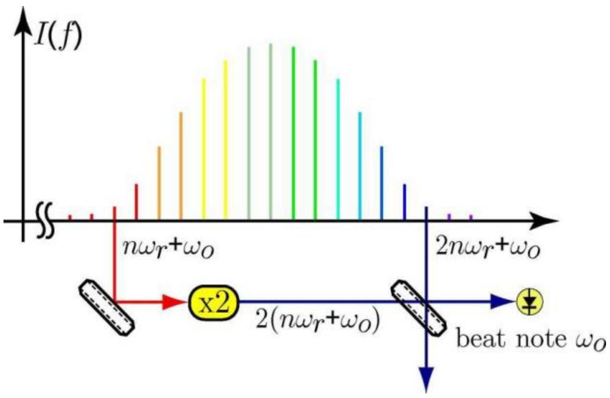

Fig 4. The principle of the optical frequency synthesizer: A mode with the mode number n at the red wing of the comb and whose frequency is given according di Eqn. (2) by $f_n = nf_r + f_o$ is frequency doubled in a non-linear crystal. If the frequency comb covers a full optical octave, a mode with the number 2n should oscillate simultaneously at $f2_n = 2nf_r + f_o$. The beat note between the frequency doubled mode and the mode at 2n yields the offset frequency $2(nf_r + f_o) - (2nf_r + f_o) = f_o$. $\omega = 2\pi f$. In other words, f_0 is detected by the self-referencing technique with an $f -2f$ interferometer setup, by taking the difference of the frequencies of a comb mode at f_n and the second harmonic of the mode at f_{2n}

EXPERIMENTAL SETUP

Optimizing the SNR of the beat signal

For the calibration of laser sources by beat measurements using a stabilized laser as standard, a beam superposition is necessary to produce the heterodyne signal. Due to the immense spectral power for both beams, producing a useful SNR (signal to noise ratio) is not very demanding. A semi-transparent mirror is sufficient in most cases.

Measurements using comb radiations on the other hand require utmost efforts to reach a useful SNR. Beside a non-Gaussian beam profile (caused by second harmonic generation) the power spread to thousands of comb modes are the main reasons for this challenge.

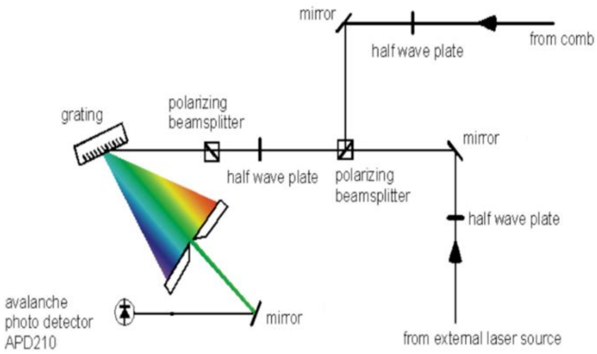

Fig 5. Beat detection unit (schematic)

The primary difficulties with measuring beat signals arise from the fact that only a very small fraction signal of the comb power is in the mode that gives rise to a beat signal. There are needs to maximize SNR and reduce detector saturation effects. Probably the most important single step is simply to take great care in aligning the beam from the test laser with the beam from the comb. It is very important to assure perfect matching of the wavefronts – two coaxial beams with the same waist position and same size overlap very well and travel accurately in the same direction to obtain a good beat signal [7, 8, 10].

The beat detection unit (Fig. 5) consists of a series of mirrors, polarized beam splitters, $\lambda/2$ waveplates, which aim to place the beams from both the laser comb and calibration in the same polarisation plane, and lead them to a photodetector. In addition to delivered beat detection unit, its set up is upgraded with long focal lenses [8] to obtain a useable SNR of beat signal. Setup for this method is shown in Fig 7.

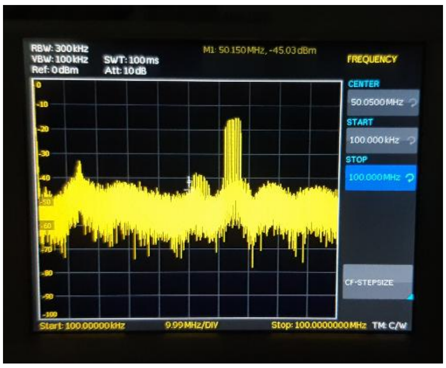

Fig 6. Beat note signal

In our case, SNR over around 30 dB (@ 300 MHz bandwidth) is sufficient and can be considered as usable for reliable measurement of beat frequencies, the more, the better. (Fig. 6)

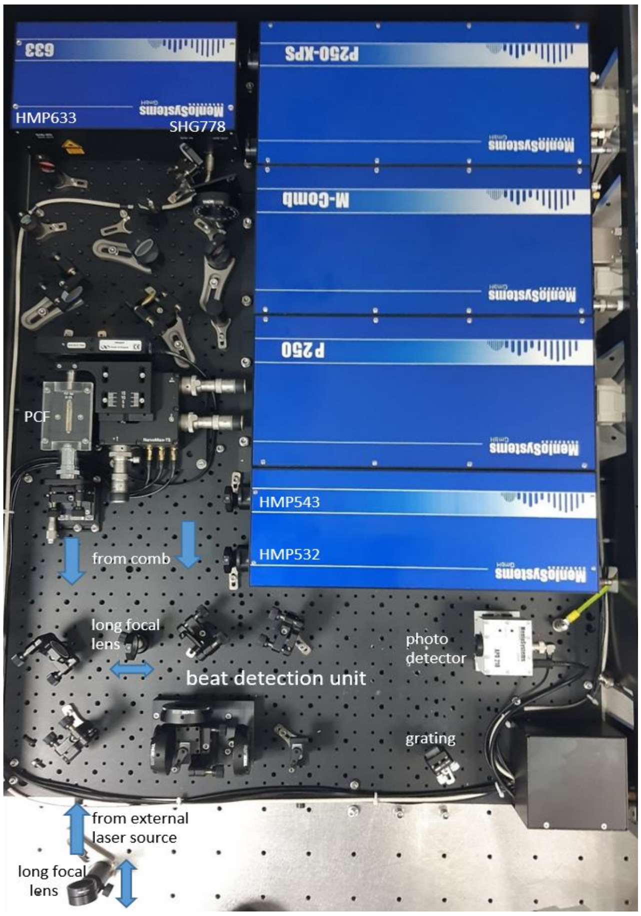

Fig 7. Measurement setup

Monitoring and counting the beat frequency

For measuring the beat note signal between the laser and comb mode, a diffraction grating for spectral filtering is used.
The beat note is detected by a fast photo detector (avalanche photodiode).
Beat frequency signal from a photo detector is monitored and analyzed by a spectrum analyser (Figure 10) and counted by a 4 channel frequency counter which is referenced to a primary frequency standard (DMDM Cs clock, 10 MHz) with a relative uncertainty of 1×10^{-13} at 24 h (averaging time).
During the process of measurement, the frequency comb system as well as remaining phase is very reliably locked. The frequency counters used for this measurements count continuously every second without dead time.
Spectrum analyser. The beat frequency is fed into the frequency counter and spectrum analyser by passing through RF band pass filter which is placed in the counting path between photo detector and these two measuring/ monitoring instruments. RF filter has a span of about 20 MHz.in the frequency range between 50 MHz and 70 MHz. To obtain measurable signal, beat frequency has to be shifted to this frequency range. The best results (most usable signal) is obtained around 64 MHz.
Frequency counter. Menlo FXM counters are not equipped with (adjustable) hysteresis, so they always show some values even in the absence of the beat signal. So it is necessary to define somehow whether the signal level is sufficient for reliable counting or not [6].
Some laboratories do measure SNR with RF spectrum analyser and remember the threshold value. Instead of this, in this procedure method learned from Austrian Metrology Institute, BEV is used: the beat frequency is measured by two counters (or, in DMDM case, by two channels of one counter FXM50) synchronously but with slightly different conditions – one input is attenuated by 3 dB BNC fixed attenuator [8].
Both counter channels show the same values if the beat signal SNR is sufficient. With decreasing amplitude of the beat signal the values start to differ and the difference is measure of counting quality. In our case, SNR over 30 dB is sufficient for counting without cycle-slips (outliers).
Max allowed difference between two counter readings is set to 1 Hz and can be changed by software. Samples (readings) which are not agree within 1 Hz are treated as wrongly counted (outliers, cycle-slips).

MEASUREMENT OF LASER FREQUENCY (WAVELENGTH) STABILITY

The Allan deviation [$\sigma_y(\tau)$] is extremely useful for characterising a frequency source because the type of phase noise present is revealed by the way in which $\sigma_y(\tau)$ depends on the sampling time, τ [9].

However, for the Allan deviation to reliably indicate the type of noise present, it is crucial that there be no dead time between the consecutive average frequency measurements used to calculate $\sigma_y(\tau)$.
To study the stability of the laser system as well as the beat note have to be used the (overlapped) Allan variance analysis. For a set of N frequency measurements, the Allan variance is defined as

$$\sigma_y^2(\tau) = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} (f_{i+1} - f_i) \tag{3}$$

where fi denotes consecutive measurements of the average frequency, averaged over a period τ.
The Allan deviation for averaging times that are integer multiples of τ, $\tau_y(m\tau)$, can then be calculated by forming a new set of N/m average frequency values from the original set of N values. The original set is subdivided into adjacent, (non) overlapping subsets. Each value of the new set of frequency values is computed by averaging the m values in each subset of the original data. This is the prerequisite to be able to specify the Allan standard deviation and the standard error for the mean for integration times other than 1 s.

ABSOLUTE FREQUENCY MEASUREMENT OF CW LASERS

The optical frequencies of cw (continuous wave) lasers are measured by observing a beat frequency, f_{beat}, with the nearest mode of the frequency comb [6, 7, 10]. Therefore, the optical frequency, f_{cw}, is expressed as

$$f_{cw} = nf_r \pm f_o \pm f_{beat} \tag{4}$$

The mode number n is determined on the assumption that the He-Ne/I_2 frequency is known a priori to better than a few megahertz. The signs of the beats f_o and f_{beat} are determined by checking the increase or decrease of f_{beat} when f_r or f_o is increased under the conditions of locking f_r or f_o, respectively.
If the visible cw lasers are measured after second harmonic generation (SHG), that is our case, equation (4) has to be modified to take the SHG process into consideration: the f_o has to be multiplied by a factor of two:

$$f_{cw} = nf_r \pm 2f_o \pm f_{beat} \tag{5}$$

The repetition rate remains unchanged due to the fact that the dominant process in the SHG is sum frequency generation.

Evaluating measurement uncertainty

According to [9] simplified approach and applying the law of propagation of uncertainty [11], the corresponding expression is obtained by using combined standard uncertainty $u_c(y)$ (assuming the absence of correlations).

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) = \sum_{i=1}^{N} [c_i u(x_i)]^2 = \sum_{i=1}^{N} u_i^2(y) \tag{6}$$

In our case, derived from model equation (5), by collecting everything and substituting, we obtain following uncertainty equation

$$u^2(f_{laser}) = u^2(comb) + u^2(f_{beat}) \tag{7}$$

Comb. The uncertainty of the frequency comb depends on the uncertainty in determining the offset frequency, f_{off} and the repetition frequency, f_{rep}. However, both frequencies depend on and are referenced to a primary frequency standard (DMDM Cs clock, 10 MHz) with a relative uncertainty of 1×10^{-13} at 24 h (averaging time).
Laser beat. The uncertainty associated with the measured value of the beat signal is estimated using a rectangular type of distribution, with an amplitude (fmax – fmin), where the first is the maximum value and the second is the minimum value of the measured beat frequencies.

$$u(f_{beat}) = \frac{f_{beat}^{max} - f_{beat}^{min}}{2\sqrt{3}} \quad (8)$$

where factor √3 in the denominator comes from the rectangular distribution we have assumed.

RESULTS AND CONCLUSION

The aim of this paper is apply the comb generator to measure the absolute frequency of an iodine-stabilized He-Ne laser at 633 nm nominal wavelength stabilized on the four components, d, e, f and g of the hyperfine transition R(127) 11-5. The obtained result was compared with the same values recommended by the CCL.

At DMDM we have several He-Ne lasers with a nominal wavelength at 633 nm stabilized to hyperfine components of transition 11-5 R(127) of $^{127}I_2$ vapour in internal cell with third harmonic locking technique. Here we present the results of absolute frequency measurements of laser DMDM1 which is in operation since 1990 with different modifications.

The results of measurements of the four components of laser DMDM1 with the comb at DMDM are in very good agreement with the CCL ones (MeP), e.g. within 20 kHz MeP expanded uncertainties.

Results are given in Table 1. The expanded measurement uncertainty for all four measured frequency components is 0,5 kHz.

Table 1. Measured frequencies of the four components referenced to the MeP values. All values in kHz.

Component	Measured frequency
d	473 612 379 828 − 14,1
e	473 612 366 967 − 11,7
f	473 612 353 604 − 11,1
g	473 612 340 406 − 9,0

Laboratories which use a light source which is part of the *CIPM List of recommended frequency standard values* for their realisation of the metre are required to take part in the international key comparison CCL-K11 [5] at least every 10 years (unless they are node laboratories in this comparison). The comparison tests the laboratory's ability to realise the relevant optical frequency standard within their stated uncertainties.

In 2011 (indirectly by transfer laser BEV-2) and 2021 (directly), the absolute frequency of the DMDM1 laser, f_f, was measured against the BEV fiber femtosecond laser comb following the technical protocol for the key comparison CCL-K11 for optical frequency/wavelength standards. The measured optical frequency, f_f , of the DMDM1 laser agreed with the CIPM recommended frequency value (MeP) within MeP expanded uncertainty of 20 kHz.

The results from those two comparison cycles are summarized in Table 2.

Table 2. Results for frequency f_f of DMDM1 laser obtained in CCL-K11 comparison in 2011 and 2021.

	2011	2021
Expected frequency f_e (kHz)	473 612 353 604,0 (12,0)	473 612 353 594,1 (12,0)
Measured frequency – uncorrected f_m (kHz)	473 612 353 593,4 (1,9)	473 612 353 592,654 (0,070)
Frequency difference $\Delta f = f_e - f_m$ (kHz)	+ 10,6	+ 1,4
Fractional frequency difference $\Delta f / f_e$	+ 22,4·10^{-12}	+ 3,1·10^{-12}
Degree of equivalence (DoE) stated as E_n value	+ 0,44	+ 0,06

Achieved improvements of the upgraded DMDM1 laser was evaluated as part of CCL-K11 comparison monitoring obtained performance of DMDM1 in 2011 and in 2021. Besides absolute frequency measurements of the laser by comb generator, the frequency stability of DMDM1 laser is measured as well by measuring the Allan variance before (2011) and after (2021) upgrade and results are shown in Fig. 8.

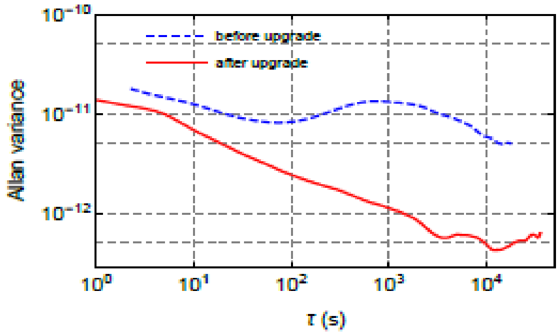

Fig. 8: Allan variance of the DMDM1 laser before and after upgrade

The optical frequency comb generator at DMDM was successfuly put into operation. DMDM developed method for the calibration of laser sources by beat measurements using a comb generator as an exstension on similar measurements established by using stabilized laser at primary level. We have measured the absolute frequency of the DMDM1 standard laser at 633 nm stabilized on the d, e, f and g components of the 11-5 R(127) hyperfine transition of the $^{127}I_2$ molecule. The result of measurements are in good agreement with the difference between the mean frequency of the four components of the standard laser and those of CCL recommended values for the same components.

It is obvious that the applied upgrade improved performances of the DMDM1 laser. Laser stability (Allan variance anlysis) is much better. Long term drift of laser frequency in the period of 10 years (between two CCL-K11 comparison) is less than 1 kHz which is excellent result for the long term frequency stability of the DMDM1 primary laser

REFERENCES

[1] BIPM 2019, The International System of Units (SI Brochure), 9th edition, https://bipm.org/en/publications/si-brochure

[2] BIPM 2019, The International System of Units (SI Brochure), 9th edition, Appendix 2: Practical realization of the definitions of some important units – mises en pratique, *Mise en Pratique for the definition of the metre in the SI*, https://www.bipm.org/utils/en/pdf/si-mep/SI-App2-metre.pdf.

[3] BIPM 2019, The International System of Units (SI Brochure), 9th edition, Appendix 2: Practical realization of the definitions of some important units – mises en pratique, *Recommended values of standard frequencies*, https://www.bipm.org/en/publications/mises-en-pratique/standard frequencies.

[4] Riehle F, Gill P, Arias F, and Robertson L, "The CIPM list of recommended frequency standard values: guidelines and procedures", *Metrologia* **55** (2018) 188, https://iopscience.iop.org/article/10.1088/1681-7575/aaa302

[5] CCL Key comparison CCL-K11: *Comparison of optical frequency and wavelength standards*, https://www.bipm.org/kcdb/comparison?id=978

[6] P. Balling, *Laser frequency stabilization and measurement of optical frequencies*, Ph.D. Thesis, Charles University Prague, Faculty of Mathematics and Physics, 2009

[7] S. Samoudi, M. mar Perez, S. Ferreira-Barragans, E. Prieto, *Absolute optical frequency measurements of iodine-stabilized He-Ne laser at 633 nm by using a femtosecond laser frequency comb*, Int. J. Metrol. Eng. 3, 101-106, 2012

[8] PTB Project in Technical Cooperation - Support of the quality Infrastructure in Serbia, *Hands-on training for the use of the Menlo frequency comb*, trainer/reporter: M. Matus (BEV), 2019

[9] B. Samoudi, *Realization of the metre by using a femtosecondlaser frequency comb: application in optical frequency metrology*, Int. J. Metrol. Qual. Eng. 8, 16, 2017

[10] W.J. Riley, *Handbook of Frequency Stability Analysis*, NIST Special publication 1065, 2008

[11] FC1500-250-WG Optical frequency synthesizer system *User Manuals*
[12] GUM: *Guide to the expression of uncertainty in measurement*, JCGM100:2008
[13] Z. Grujić, M. Nikolić, S. Zelenika, M. Rabasović, *An upgrade of the primary length standard of Republic of Serbia where digital stabilization is performed by Arduino Due board,* Rev. of Sci. Instr., Issue 2, Vol.1995, 2024